International Journal of Theoretical Physics, Vol. 38, No. 4, 1999

The Fluid of Primordial Fluctuations

Mariana Graña[1]

Received September 15, 1998

We formulate a phenomenological model where the inflaton fluctuations are treated as a fluid. By applying the hydrodynamic equations to this fluid we recover the conventional result that relates the spectrum of density fluctuations in the inflaton field at reentering the horizon to the spectrum of fluctuations at the time a scale leaves the horizon. Moreover, through the equivalent viscosity of the fluid we obtain a Reynolds number that suggests turbulent motion, which implies that mode–mode coupling in the inflaton field cannot be neglected. For de Sitter inflation the resulting spectrum using turbulence theory is scale invariant on all scales of interest. This suggests that the hypothesis of an extremely weakly coupled inflation could be relaxed without affecting the predictions of the model.

1. INTRODUCTION

Inflationary models were originally introduced as a solution for the so-called puzzles of standard hot big bang cosmology [1], namely the horizon, flatness, and photon-to-baryon ratio problem [2, 3]. Soon after the original proposal, it was realized that inflation could perform a subtler task: to provide a framework for explaining the origin of primordial density fluctuations [4]. Quantum fluctuations of the inflaton field distort the reheating surface, inducing a primordial density contrast (see ref. 1; we shall review this argument in greater detail below) $\delta\rho/\rho \sim (H/\phi)\delta\phi$, H being the inflationary Hubble parameter. All quantities on the right-hand side of this relation are evaluated as the relevant mode leaves the horizon. In order to obtain a concrete prediction from this equation we must estimate the quantum fluctuations $\delta\phi$. The usual approach treats these fluctuations as a free field (for example, in the seminal paper by Starobinsky [5]) in its (de Sitter-invariant) vacuum state or very close to it [6]. Then a simple calculation within quantum field theory in curved spaces allows us to evaluate the $\delta\phi$ as $\delta\phi \sim H$ [7].

[1] IAFE and Physics Department, University of Buenos Aires, Argentina.

0020-7748/99/0400-135 9$16.00/0

The main goal of this paper is the application of hydrodynamics to describe the macroscopic behavior of quantum fluctuations. This is possible because these fluctuations, as far as they are relevant to our present concerns, may be described by a c-number energy-momentum tensor subject both to the usual conservation laws and the second law of thermodynamics [7]. There is therefore an equivalent fluid description, consisting of a classical fluid whose energy-momentum tensor and equation of state reproduce the observed ones for the quantum fluctuations. Solving the dynamics of this equivalent fluid yields answers to all relevant questions concerning the behavior of the actual quantum fluctuations. As we shall show below, conditions in the early stages of inflation are such that, for generic initial conditions, the flow is highly turbulent, meaning that the corresponding Reynolds number is well over 1000. This description will then as well enable us to develop ways of estimating the primordial density contrast in chaotic inflation models without presupposing that couplings among fluctuations are negligible, which will be done by investigating the turbulent regime of the fluid.

The underlying physics in our model is the same as in these more familiar approaches to primordial fluctuation generation: we do not question the ultimate quantum origin of the fluctuations, but borrow insights from hydrodynamics to describe the macroscopic behavior of these fluctuations, rather than rely on possibly oversimplified linearized microscopic models. The conditions of validity of our procedure are the assumptions that the energy momentum tensor of fluctuations is a c-number quantity (which ought to be true at any scale below Planck's) and the second law.

An immediate consequence of energy-momentum conservation and the second law is that when velocities are low, the phenomenological fluid may be described within the Eckart spacetime Navier–Stokes equation. The model is then defined by giving the equation of state and the viscosity of the equivalent fluid. The advantage we gain is that these are features that can be computed locally. As far as the relevant scales are much below the curvature radius, it is possible to use for them their standard flat-spacetime values. At high temperatures we obtain the equation of state for radiation, $p = (1/3)\rho$, and a dynamic viscosity $\eta \sim T^3$. Since the speed of sound, being close to the light speed, is much higher than the characteristic speed of the fluid, the flow may be considered incompressible. There will be fluctuations in velocity, nevertheless, and these are the ones responsible for density fluctuations, as is well known [1].

Once the equivalent fluid description is set up, the task at hand to work on the turbulent regime is to study the evolution of a typical eddy as it is blown up by the universal expansion, exchanging and dissipating energy while inside the horizon, and freezing when outside, until it reaches the reheating hypersurface and delivers its energy to radiation. By assuming that the turbulent velocity fluctuations in the eddy produce fluctuations in the energy density of radiation in the usual way, we shall be able to relate the primordial density contrast to the features of the original self-similar turbulence. The resulting spectrum may be matched against the known data on the cosmic microwave background [10], providing a crucial test of the inner consistency and viability of the approach. Our conclusion is that, insofar as the horizon remains constant during inflation, the spectrum of primordial density fluctuations produced by self-similar flows is strictly scale invariant ($n = 1$; see ref. 11) at large scales. Quantitative agreement with observations may be obtained without any special fine tuning.

The rest of the paper is organized as follows. In the next section we provide a brief summary of hydrodynamics in expanding universes, in order to set up the language for the rest of the paper. In Section 3 we proceed to discuss the equivalent fluid description of inflaton fluctuations, and how to extract the primordial density contrast therefrom. As a simple application of the method, we consider briefly the case of free fluctuations, showing that the model leads back to the conventional results. In Section 4 we present Chandrasekhar's self-similar solutions and their generalization to expanding universes, deriving the corresponding scale-invariant primordial contrast. We state our main conclusions in the final section.

2. HYDRODYNAMIC FLOWS IN EXPANDING UNIVERSES

For a curved spacetime, in particular that described by a Friedmann–Robertson–Walker (FRW) background metric with zero spatial curvature [$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$], the generalization of the hydrodynamic equations has been considered by many authors [12–16]. We follow Tomita *et al.*'s analysis [17], in which they obtain the solution for the energy spectrum in the case of homogeneous, isotropic, and incompressible turbulence.

In a generic spacetime, we describe fluid flow from the energy density ρ, pressure p, and four-velocity U. The symmetries of the FRW solution suggest using instead the commoving three-velocity $u^i = U^i/U^0$; if $U^i \ll U^0$ the flow is nonrelativistic, and if $\nabla \mathbf{u} = 0$, it is incompressible [$\mathbf{u} = (u^1, u^2, u^3)$]. Later we shall also use the physical three-velocity $v = a(t)u$.

The corresponding continuity and Navier–Stokes equations for a Robertson–Walker background are obtained by the condition of conservation of the energy-momentum tensor [8].

To analyze the system's behavior, we define the two-point, one-time correlation function for the velocity:

$$R_{ij}(\mathbf{x}, \mathbf{x}', t) = a^2 \langle u_i(\mathbf{x}, t) u_j(\mathbf{x}', t) \rangle \qquad (1)$$

Let us also define the following correlation tensors:

$$P_{ij}(r, t) = \frac{1}{(p + \rho)} \left(\frac{\partial}{\partial r_i} \langle p(\mathbf{x}, t) u_j(\mathbf{x}', t) \rangle - \frac{\partial}{\partial r_j} \langle p(\mathbf{x}', t) u_i(\mathbf{x}, t) \rangle \right) \quad (2)$$

and

$$T_{ij}(r, t) = a^2 \frac{\partial}{\partial r_k} \langle u_i(\mathbf{x}, t) u_k(\mathbf{x}, t) u_j(\mathbf{x}', t) - u_i(\mathbf{x}, t) u_k(\mathbf{x}', t) u_j(\mathbf{x}', t) \rangle \quad (3)$$

If we call Φ_{ij} the Fourier transform—done in terms of comoving wave-numbers—of R_{ij}, and Γ_{ij} those corresponding to T_{ij}, we obtain the equation for the energy spectrum for a fluid with shear viscosity $\eta = \nu\,(p + \rho)$ and zero bulk viscosity, namely

$$-\frac{\partial}{\partial t} E(k, t) = T(k, t) + 2\left\{ \frac{\nu k^2}{a^2} + \frac{\partial \ln((p + \rho) a^4)}{\partial t} \right\} E(k, t) \quad (4)$$

where

$$E(k, t) = \frac{1}{2} \int \Phi_{ii}(\mathbf{k}, t) k^2 \, d\Omega(\mathbf{k}); \qquad T(k, t) = -\frac{1}{2} \int \Gamma_{ii}(\mathbf{k}, t) k^2 \, d\Omega(\mathbf{k}) \quad (5)$$

The inertia term $T(k, t)$ is the one that contains the mode–mode interaction, and its effect is to drain energy from the more energetic modes—typically the bigger ones—to the ones where there is major viscous dissipation—the smaller ones.

3. EQUIVALENT FLUID FOR INFLATON FLUCTUATIONS

After establishing the basic necessary notions for the description of hydrodynamic flows, our goal is to associate an equivalent fluid description to inflaton fluctuations, and to derive the spectrum of primordial density fluctuations at reheating therefrom. We shall discuss in the following sections some nontrivial instances of this method.

3.1. The Inflaton as a Fluid

To describe the inflaton field from the point of view of an equivalent fluid, we need to obtain the energy density, pressure, and velocity of this fluid as functionals of the state of the field. To this end, our starting point will be that in the rest frame of the fluid (quantities in this frame being labeled by a curl), the field ought to be spatially constant

$$\nabla \phi = 0 \quad (6)$$

To obtain the fluid four-velocity, we make a boost to the comoving frame. Then, the boost's characteristic velocity will be the one we are seeking. By the condition (6) we obtain

$$u_i = -\frac{\partial_i \phi}{\phi} \tag{7}$$

which is generalized to the covariant form

$$u_\mu = -\frac{\partial_\mu \phi}{\sqrt{-\partial_\rho \phi \partial^\rho \phi}} \tag{8}$$

The energy density in the rest frame must be

$$\tilde{\rho} = \frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2 + V(\phi)$$

Using the Lorentz transformations with the four-velocity (8), we obtain the general form for the energy density

$$\rho = -\frac{1}{2}\,\partial_\rho \phi \partial^\rho \phi + V(\phi) \tag{9}$$

Finally, we obtain the pressure imposing an equality between the energy-momentum tensor for a perfect fluid (see, for example, ref. 8) and that for a minimally coupled scalar field (see ref. 7). The resulting pressure is

$$p = -\frac{1}{2}\partial_\rho \phi \partial^\rho \phi - V(\phi) \tag{10}$$

Since the possibility of deriving a Navier–Stokes equation for the equivalent fluid rests on the conservation of $T_{\mu\nu}$, in principle only the whole inflaton field can be thus represented. However, under the approximation that the homogeneous part of the inflaton essentially contributes an effective cosmological constant, the background energy-momentum tensor $T_{0\mu\nu} = \Lambda g_{\mu\nu}$ is independently conserved (even if Λ were not constant, conservation fails only on scales too large to be cosmologically relevant), and we can associate an equivalent fluid to the inhomogeneous quantum fluctuations $\delta\phi$ alone. For this fluid, we find the physical velocity ($v^i = au^i = a^{-1}u_i$)

$$v_k^i = k_{phys}^i \left(\frac{\delta \phi_k}{\phi_0}\right) \tag{11}$$

where ϕ_0 is the homogeneous background. We interpret this equation to mean that stochastic averages of the fluid velocity are to be identified with (symmetric) quantum expectation values of the operator on the right-hand side [18, 19]. As far as the equation of state is concerned, the free energy for a massive scalar field in the high-temperature limit ($T \gg m$) [20] gives us the relationship between the pressure and the energy density, which turns out to be that for radiation, $p = (1/3)\rho$. This means that the energy density for this fluid redshifts proportional to a^{-4}. As this result has been obtained for a flat space time, it is valid for scales smaller than the curvature radius. When scales are bigger than the Hubble radius, which takes place when the high-temperature limit is no longer valid, the velocity u_i [Eq. (7)] must remain constant, which in turn means that the physical three-velocity $v^i = av^i$ must redshift proportional to a^{-1}. As these scales are frozen out because they are outside the horizon, they cannot interact among them or be dissipated by viscosity. Thus, the Navier–Stokes equation for a Robertson–Walker background reduces to

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{\partial \ln((p + \rho)a^4)}{\partial t}\mathbf{v} = 0 \tag{12}$$

We obtain $v \propto a^{-1}$ when $(p + \rho) \propto a^{-3}$, corresponding to the equation of state of matter: $p = 0$. Thus, when the scales are well outside the horizon, our fluid behaves as pressureless dust, in agreement with the well-known prediction based on the virial theorem for the equation of state of a field undergoing oscillations [21], which occurs at the final period of inflation. We must point out that the hypothesis of incompressibility is no longer valid for an equation of state of this type. Nevertheless, for scales bigger than the Hubble radius, which cannot decay through nonlinear interaction or dissipation by viscosity, Eq. (12) is still valid, regardless of the ratio of typical velocities to the speed of sound.

3.2. Transport Coefficients

The framework to obtain transport coefficients for our fluid is linear response theory. In the limit of slow variations in space and time of the magnitudes involved in the equation of conservation for the energy-momentum tensor, the system's response while it is slightly displaced from equilibrium can be alternatively described by Navier–Stokes and continuity equations as well as by equilibrium expectation values of correlation functions. Matching these two descriptions, one obtains the Kubo formula for the shear viscosity [22]:

$$\eta = \frac{1}{6}\lim_{w,k\to 0}\left[\frac{1}{w}\int dt\int d^3\mathbf{r}\, e^{i(\mathbf{k}\cdot\mathbf{r}-wt)}\langle[\pi_{ij}(\mathbf{r},t),\, \pi^{ij}(\mathbf{O},0)]\rangle_{\text{eq}}\right] \tag{13}$$

where π_{ij} are the traceless spatial–spatial components of the energy-momentum tensor:

$$\pi_{ij} = T_{ij} - \frac{1}{3}\delta_{ij}T_k^k$$

For a minimally coupled scalar field the shear viscosity in the high-temperature limit ($T \gg m, \Gamma$) turns out to be

$$\eta = const \cdot T\Gamma^2 \tag{14}$$

The thermal width Γ comes from the imaginary part of the self-energy. From dimensional analysis, it must be proportional to the temperature since the only relevant scale in the high-temperature limit is the temperature itself (the constant of proportionality must be much less than unity for the consistency of the high-temperature limit). Assuming that the only present interaction is the one coming from a $\sigma\phi^4$ term, Γ must include two σ insertions, which means that Γ must be proportional to σ^2 (we will estimate it as $\Gamma \sim \sigma^2 T$). The shear dynamic viscosity becomes

$$\eta \sim \sigma^4 T^3 \tag{15}$$

A similar analysis allows us to evaluate the bulk viscosity, which turns out to be zero for a fluid with an equation of state of the type $p = (1/3)\rho$ [23], in agreement with our previous assumptions.

3.3. Conversion of Hydrodynamic Fluctuations into Primordial Density Contrast

Having described the quantum field as a fluid, we will analyze the resulting spectrum of density inhomogeneities. To do so, we assume that at some time t_1 during the beginning of the inflationary phase, when all the scales relevant to cosmology were inside the horizon, the scalar field fluctuations were undergoing hydrodynamic fluctuations. Once the scales leave the Hubble radius, their energy cannot be dissipated by viscosity or by nonlinear coupling. Thus, Eq. (4) means that they evolve according to

$$E(k, t > t_{\text{out}}) = E(k, t = t_{\text{out}})\left[\frac{((p + \rho)a^4)_{\text{out}}}{((p + \rho)a^4)(t)}\right]^2 \tag{16}$$

where the subscript "out" refers to the time when each scale leaves the horizon.

The definition of $E(k)$, Eq. (5), can be written in terms of the Fourier transform of the velocity; since the flow is statistically isotropic and homogeneous,

$$\langle v^i(\mathbf{k})v^i(\mathbf{k}')\rangle = \left(\frac{E(k)}{4\pi k^2}\right)\delta^3(\mathbf{k} + \mathbf{k}') \tag{17}$$

Combining (16) and (17), we can obtain the r.m.s. value for the scalar field velocity at the time of reheating, which will be the r.m.s. value of the perturbation in the radiation's streaming velocity. This perturbation will in turn produce fluctuations in the energy density of radiation, which will evolve in the usual way. The theory of relativistic very large wavelength fluctuations predicts $\delta \sim a^2$, where $\delta = \delta\rho/\rho$ is the density contrast, and thus $\dot{\delta} \sim H\delta$, while the continuity equation yields $\dot{\delta} \sim v/l$ on a scale of physical size l. Consistency of these two pictures leads to the relationship between the velocity at the time of reheating and the fluctuation in the energy density as

$$\left.\frac{\delta\rho}{\rho}\right|_{\mathrm{reh}} = \frac{v_{\mathrm{reh}}}{lH_{\mathrm{reh}}} \tag{18}$$

where H_{reh} is the Hubble parameter at reheating. Following these fluctuations up to the time they reenter the Hubble radius, assuming that their size is such that they are always unstable (they must be always bigger than the Jeans length), they grow following the law (see, for example, refs. 1 and 8)

$$\delta \equiv \left.\frac{\delta\rho}{\rho}\right|_{\mathrm{ent}} = \left.\frac{\delta\rho}{\rho}\right|_{\mathrm{reh}} \left(\frac{a_{\mathrm{eq}}}{a_{\mathrm{reh}}}\right)^2 \left(\frac{a_{\mathrm{ent}}}{a_{\mathrm{eq}}}\right) \equiv H_{\mathrm{reh}}^2 l^2 \left.\frac{\delta\rho}{\rho}\right|_{\mathrm{reh}} \tag{19}$$

where the subscript "ent" means the time each scale reenters the Hubble radius (the second equation holds even if the entering time occurs before matter–radiation equality). Combining equations (18)–(19), we obtain the density contrast predicted by this theory at the time the modes reenter the Hubble radius:

$$\langle \delta_k \delta_{k'} \rangle_{ent} = \frac{H_{reh}^2 a_{reh}^2 E(k, t = t_{reh})}{4\pi k^4} \delta^3(\mathbf{k} + \mathbf{k}') \tag{20}$$

This is the main result of this paper, as it relates the density contrast to a hydrodynamic variable. We shall see a nontrivial application of this formula in the next section, but first it is convenient that we pause to show explicitly how the familiar results relating to free field fluctuations are recovered in this language. Probably the most important feature of a theory where inflaton fluctuations are free is that each mode evolves independently of the others. Immediately after leaving the horizon they freeze, a situation that can be described phenomenologically by assigning to the mode the effective equation of state of dust. This implies that

$$E(k, t = t_{reh}) = E(k, t = t_{out}) \left(\frac{a(t = t_{out})}{a(t_{reh})}\right)^2 \tag{21}$$

$$\langle \delta_k \delta_{k'} \rangle_{ent} = \frac{E(k, t = t_{out}(k))}{4\pi k^2} \delta^3(\mathbf{k} + \mathbf{k}') \tag{22}$$

Let us compare this expression to the usual one in terms of quantum fluctuations. First we use Eq. (17), neglecting any variation of H or of the velocities during reheating, to get

$$\langle \delta_k \delta_{k'} \rangle_{ent} = \langle v^i(\mathbf{k}) v^i(\mathbf{k})' \rangle \rangle|_{t=t_{out}(k)} \tag{23}$$

We now relate the physical velocity to field fluctuations according to Eq. (11); at $t = t_{out}(k)$, $k_{phys} = H$, and this reduces to

$$\langle \delta_k \delta_{k'} \rangle_{ent} = \left(\frac{H}{\dot{\phi}} \right)^2 \langle \delta\phi_k \delta\phi_{k'} \rangle|_{t=t_{out}(k)} \tag{24}$$

which is the conventional result [24].

This shows the agreement between the fluid description and the conventional approach in this case.

4. SELF-SIMILAR FLOWS AND NONLINEAR FLUCTUATIONS

In the previous sections we set up the general formalism whereby we can associate to the evolution of quantum fluctuations during inflation an equivalent fluid description, and derive the corresponding primordial density contrast from hydrodynamic variables. Of course, to put the formalism to actual use, we must be able to solve Navier–Stokes equations, which is in itself almost as daunting as solving the fundamental quantum field theory. However, there is in the hydrodynamic case a century of lore to draw upon [25], and some well-tested approximations leading to relatively simple solutions. In this section, we shall demonstrate the equivalent fluid method by investigating the spectra resulting from one of these solutions, namely self-similar flows. Toward the end of the section, we shall discuss the relevance of these solutions to actual cosmology.

As we have seen in the previous section, Eq. (20), the key element in deriving the primordial density contrast is the energy spectrum $E(k)$, Eq. (5), which is the solution of the balance equation (4). By providing closure, that is, writing the inertial force in terms of the spectrum itself, which was done by Heisenberg [26], a closed evolution equation for $E(k)$ is obtained. Chandrasekhar [27] solved this equation for decaying turbulence, assuming that there is a stage in the decay where the bigger eddies have a sufficient amount of energy to maintain an equilibrium distribution, thus requiring that the solution for the spectrum should be self-similar. These solutions were generalized to flows in expanding universes by Tomita *et al.* [17], obtaining a linear spectrum for scales much bigger than the Taylor microscale λ:

$$E(k, t) = 4v_{ti}^2\left(\frac{(p+\rho)_i a_i^4}{(p+\rho)a^4}\right)^2 \lambda_i^2 k \qquad \text{for} \qquad \lambda k \ll 1 \tag{25}$$

where the Taylor microscale λ and the turbulent velocity v_t are defined by

$$\lambda^2(t) \equiv 5\frac{\int E(k, t)\, dk}{\int E(k, t)k^2\, dk}, \qquad \frac{1}{2}v_t^2(t) \equiv \int E(k, t)\, dk \tag{26}$$

and they must follow the law

$$\lambda^2(t) = \lambda_i^2 + 10\int_{t_i}^{t} \frac{\eta}{(p+\rho)a^2}\, dt, \qquad v_t = v_{ti}\left(\frac{(p+\rho)_i a_i^4}{(p+\rho)a^4}\right)\frac{\lambda_i}{\lambda(t)} \tag{27}$$

We now want to place a self-similar solution in the context of an inflationary scenario where, instead of regarding the inflaton fluctuations as free, we shall replace them by an equivalent fluid, whose evolution we will assume to be self-similar.

We will assume a duration of inflation close to the minimum value ($N_{\min} \simeq 60$, where N stands for the number of *e-folds*), which can be justified by the expected quadrupole anisotropy [28] as well as by the ratio of the present to the critical density. By this assumption, a scale whose present size equals the horizon ($\simeq$ 3000 Mpc) leaves the Hubble radius soon after the beginning of inflation.

Unless we are in the free field case, here we cannot deal with each mode independently, but we must treat the whole flow subject to a phenomenological equation of state. Let us assume the self-similar flow sets in at a time t_1 when the temperature $T \gg H$, and that the present horizon scale leaves the horizon at or around time t_1. Then it is valid to use the high-temperature limit for length scales close to the present horizon while they leave the Hubble radius during the inflationary phase. The fluid's equation of state in this limit is of the $p = \frac{1}{3}\rho$ type, which means that the product $(p+\rho)a^4$ remains constant throughout the universal expansion. The factor $((p+\rho)a^4)_{out}$ involved in (16) is then independent of the particular scale being considered within this group. Thus, using (16) and (25) we can obtain the energy spectrum for these scales while they are outside the horizon:

$$\frac{E(k, t > t_{out}(k))}{E(k_0, t > t_{out}(k_0))} = \frac{E(k, t = t_{out}(k))}{E(k_0, t = t_{out}(k_0))} \tag{28}$$

where [cf. (25)]

$$E(k, t = t_{out}) = 4v_t^2(t_1)\lambda^2(t_1)k \tag{29}$$

$\lambda(t_1)$ is the comoving Taylor microscale at the time the self similar flow sets in. As at the initial time t_1 the only relevant scale is the temperature, we expect the initial Taylor microscale to be the inverse of the temperature at that time, i.e., $\lambda_{phys}(t_1) \sim 1/T(t_1)$. For a viscosity given by (15), using (27)–(29), the resulting spectrum for the scales at the time of reentering the Hubble radius turns out to be

$$\langle\delta_k\delta_{k'}\rangle_{ent} = \frac{v_t^2(t_1)}{\pi}\frac{1}{k^3}\delta^3(\mathbf{k} + \mathbf{k}') \tag{30}$$

that is, a scale-invariant Harrison–Zel'dovich spectrum [29] with amplitude v_t. The constraint of $k \ll \lambda^{-1}$ reduces to a minimum scale above which we obtain scale invariance. Nevertheless, the Jeans length imposes a lower limit bigger than this (we are assuming that the scales are always unstable while they are outside the horizon, which is valid if they are bigger than the Jeans length).

Finally, combining the estimates for the initial Taylor microscale and the turbulent velocity, we obtain a Reynolds number:

$$R = \frac{4}{3}\frac{\lambda_{phys}(t_1)v_t(t_1)\rho(t_1)}{\eta(t_1)} \sim \frac{v_t(t_1)}{\sigma^4}$$

suggesting highly turbulent motion, especially for small couplings.

5. FINAL REMARKS

In this paper we developed an alternative hydrodynamic description for fluctuations in the inflaton field.

In present conventional approaches, density inhomogeneities arise from primordial fluctuations in the inflaton field, ultimately of quantum origin. Fluctuations are treated as a free field, thus forcing upon us the assumption that higher derivatives of the inflaton potential are negligible. Through the equivalent fluid description we sought a direct estimate of the primordial density contrast generated in a nonlinear inflationary model. Our model successfully reproduces the results of the conventional approach on very large scales, that is, a scale-invariant spectrum.

In our view, one result of our work is not that a self-similar solution should be our final description of fluctuations during inflation, but rather that it is possible to make sense of the physics of fluctuations even in rather general potentials. The self-similar solutions we have explored in some detail should be seen as an ideal case which will more or less approximate actual flow patterns; indeed, the same could be said of the de Sitter-invariant vacuum as a description of the actual state of the field in free theories.

The connection of the physics of primordial fluctuations to hydrodynamics has some interest of its own, as it provides an alternative to brute-force quantum field-theoretic calculations, and also yields physical insight into the macroscopic behavior of quantum fields in the early universe. The equivalent fluid method opens up a wealth of new interesting phenomena, such as intermittency in the primordial spectrum [30, 31] and Burgers turbulence [32], with a strong potential impact on our understanding of the evolution of cosmic structures. Moreover, it is appealing to be able to account for a macroscopic phenomenon, such as fluctuation generation on super horizon scales, mostly on macroscopic terms (for an independent attempt in this direction, see ref. 33). We will continue our research in this field, which promises a most rewarding dialogue between cosmology, astrophysics, and nonlinear physics at large.

ACKNOWLEDGMENTS

We are grateful to C. Ferro Fontán and F. Minotti for pointing out some references to us, and to D. Gómez for many conversations on turbulence theory.

This work has been partially supported by the European project CI1-CT94-0004 and by Universidad de Buenos Aires, CONICET, and Fundación Antorchas.

REFERENCES

[1] P. J. E. Peebles, *Principles of Physical Cosmology*, Princeton University Press, Princeton, New Jersey (1993).
[2] A. Guth, *Phys. Rev. D* **23**, 347 (1981).
[3] L. Abbott and S. Pi, *Inflationary Cosmology*, World Scientific, Singapore (1986).
[4] S. Hawking, G. Gibbons, and S. Siklos (eds.), *The Very Early Universe*, Cambridge University Press, Cambridge (1983).
[5] A. A. Starobinsky, In *Field Theory, Quantum Gravity and Strings*, H. J. de Vega and N. Sánchez, eds., Springer, Berlin (1986).
[6] A. Vilenkin and L. Ford, *Phys. Rev. D* **26**, 1231 (1982); E. Mottola, *Phys. Rev. D* **31**, 754 (1985); B. Allen, *Phys. Rev. D* **32**, 3136 (1985).
[7] N. Birrell and P. Davies, *Quantum Field in Curved Space*, Cambridge University Press, Cambridge (1982).
[8] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, Wiley, New York (1972).
[9] B.-L. Hu, *Phys. Lett.* **90A**, 375 (1982); **97A**, 368 (1983).
[10] G. Hinshaw *et al.*, *Ap. J.* **464**, L17 (1996); E. L. Wright *et al.*, *Ap. J.* **464**, L21 (1996).

[11] E. R. Harrison, *Phys. Rev. D* **1**, 2726 (1970); Y. B. Zel'dovich, *Mon. Not. R. Astron. Soc.* **160**, 1 (1972).
[12] C. F. von Weizsäcker, *Ap. J.* **114**, 166 (1951).
[13] H. Nariai, *Sci. Rep. Tohoku Univ. Ser. I* **39**, 213 (1956).
[14] H. Sato, T. Matsuda, and H. Takeda, *Prog. Theor. Phys.* **43**, 1115 (1970).
[15] L. M. Ozernoi and G. V. Chibisov, *Sov. Astron.-AJ* **14**, 615 (1971).
[16] H. Nariai and K. Tanabe, *Prog. Theor. Phys.* **60**, 1583 (1978).
[17] K. Tomita, H. Nariai, H. Sato, T. Matsuda, and H. Takeda, *Prog. Theor. Phys.* **43**, 1511 (1970).
[18] L. Landau, E. Lifshitz, and L. Pitaevsky, *Statistical Physics*, Vol. I, Pergamon Press, Oxford (1959).
[19] E. Calzetta and B. L. Hu, *Phys. Rev. D* **49**, 6636 (1994).
[20] P. Ramond, *Field Theory, a Modern Primer*, 2nd ed., Addison-Wesley (1988).
[21] A. A. Starobinsky, *Phys. Lett. B* **91**, 99 (1980).
[22] L. P. Kadanoff and P. C. Martin, *Ann. Phys.* **24**, 419 (1963).
[23] S. Jeon and L. G. Yaffe, *Phys. Rev. D* **53**, 5799 (1996).
[24] A. Guth and S.-Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982).
[25] H. Lamb, *Hydrodynamics* , Dover, New York (1945).
[26] W. Heisenberg, *Proc. R. Soc. Lond. A* **195**, 402 (1948).
[27] S. Chandrasekhar, *Proc. R. Soc. Lond. A* **200**, 20 (1949).
[28] L. P. Grishchuck, *Phys. Rev. D* **45**, 4717 (1992).
[29] E. Bertshinger, In *New Insights into the Universe*, V. Martínez, M. Portilla, and D. Sáez, eds., Springer-Verlag, Berlin (1992).
[30] U. Frisch, *Turbulence*, Cambridge University Press, Cambridge (1995).
[31] U. Frisch, In *Chaotic Behavior of Deterministic Systems—Les Houches, 1981*, Course 13, G. Ioos, R. H. Helleman, and R. Stora, eds. (1983).
[32] S. F. Shandarin and Ya. B. Zel'dovich, *Rev. Mod. Phys.* **61**, 185 (1989); S. Kida, *J. Fluid Mech.* **93**, 337 (1979).
[33] W. Zimdahl, *Phys. Rev. D* **48**, 2431 (1993).